

01

JUGS

Heuristics to highlight:

- ☑ Act it out
- ☑ Draw a diagram
- ☑ Guess-and-check
- ☑ Use equations/algebra

You are given two jugs.

One holds 5 litres of water when full and the other holds 3 litres of water when full.

There are no markings on either jug and the cross-section of each jug is not uniform. Show how to measure out exactly 4 litres of water from a fountain.

Show also the following:

- i. Get 2 litres from 3 litre and 7 litre jugs.
- ii. Get 6 litres from 12 litre and 16 litre jugs.
- iii. Get 12 litres from 18 litre and 24 litre jugs.



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SUITABLE HINTS FOR POLYA STAGES I, II AND IV



I) UNDERSTAND THE PROBLEM

(c) Write down the heuristics you used to understand the problem.



Act it out and Draw a diagram - of the water level in the jugs at each step. Restate the problem in another way – Jug must be completely empty before it is filled and each time water is poured into a jug, the jug must be filled to the rim.



II) DEVISE A PLAN

(a) Write down the key concepts that might be involved in solving the problem.
Divisibility.

(c) Write out each plan concisely and clearly.



Plan 1 Aim for sub-goals – work on each part of the problem one after the other.
Act it out and Draw a diagram - of the water level in the jugs at each step until 4 litres is obtained.



Plan 2 Use equations/algebra for (ii) – express the number of times one jug is filled as X and the number of times the other jug is emptied as Y.

IV) CHECK AND EXPAND

(a) Write down how you checked your solution.



By explaining the procedure to a group member. By trying other values for the jugs and the amount required.

(b) Write down a sketch of any alternative solution(s) that you can think of.

Work until a factor of the amount required is obtained (for example 2l out of the required 4l). Then repeat the procedure the necessary number of times to obtain the required amount (in the example, twice).

(c) Give at least one adaptation, extension or generalisation of the problem.

Adaptation

You are given two jugs, one holds 5 litres of water when full and the other holds 3 litres of water when full. The cross-section of each jug is not uniform. There are no markings on the 5 litre jug but the halfway mark on the 3 litre jug is indicated. Show how to measure out exactly 3.5 litres of water from a fountain.



Generalisation

You are given two jugs, one holds m litres of water when full and the other holds n litres of water when full. The cross-section of each jug is not uniform and there are no markings on both jugs. Show how to measure out exactly k litres of water from a fountain. What amounts of water cannot be measured out?

Extension

You are given three jugs, holding a , b , and c litres of water when full respectively. The cross-section of each jug is not uniform and there are no markings on any of them. Show how to measure out exactly k litres of water from a fountain. What amounts of water cannot be measured out?

SOLUTIONS AND ASSESSMENT NOTES

SOLUTION #1

Let the number of times the 5/ jug is filled be x and the number of times the 3/ jug is filled be y . Note that x and y must be integers since there are no markings on the jugs. Note also that negative values for x or y implies 'emptied' $|x|$ times. Thus, $5x + 3y = 4$. By inspection, $x = -1$ and $y = 3$ is a solution. (Note that there are infinite number of solutions.)

Hence the 3/ jug must be filled 3 times and the 5/ jug must be poured out 1 time. Explicitly: fill the 3/ jug, pour into 5/ jug, fill 3/ jug (2nd time), pour 2/ into 5/ jug, empty 5/ jug, pour 1/ into 5/ jug, fill 3/ jug (3rd time), pour into 5/ jug to obtain 4/.

I) Get 2 litres from 3 litre and 7 litre Jugs.



Use the equation $3x + 7y = 2$. By inspection, $x = 3$ and $y = -1$ is a solution. Hence the 3/ jug must be filled 3 times and the 7/ jug must be poured out 1 time. Explicitly: fill the 3/ jug, pour into 7/ jug, fill 3/ jug (2nd time), pour into 7/ jug, fill 3/ jug (3rd time), pour 1/ into 7/ jug to obtain 2/, (empty 7/ jug).

SOLUTION #2

II) Get 6 litres from 12 litre and 16 litre jugs.



Use the equation $12x + 16y = 6$. Dividing by 4 throughout, we obtain $3x + 4y = 3/2$. Since the right hand side is not an integer, there are no integer solutions for x and y . Thus, the required 6 litres cannot be obtained.

SOLUTION #3

III) Get 12 litres from 18 litre and 24 litre jugs.



Use the equation $18x + 24y = 12$. Dividing by 6 throughout, we obtain $3x + 4y = 2$. (Note the connection between the problem and getting 2 litres from 3litre and 4 litre jugs.) By inspection, $x = 2$ and $y = -1$ is a solution. Hence the 18/ jug must be filled 2 times and the 24/ jug must be poured out 1 time. Explicitly: fill the 18/ jug, pour into 24/ jug, fill 18/ jug (2nd time), pour 6/ into 24/ jug to obtain 12/, (empty 24/ jug).

POSSIBLE STUDENT RESPONSES

Happy to be able to solve the initial part – praise and encourage the student.

Stuck at (ii) – ask if they think it is possible. If it is not, can they try to prove their assertion?

Assessment notes

Any systematic word or diagrammatic explanation for the possible situations that is clear is acceptable. A mathematical explanation involving divisibility is needed for (ii) to be accepted. If (ii) is not acceptably solved, mark the solution as partially correct.